I. SOLUTION BY GEO. W. HARTWELL, Hamline University.

Let

$$S = 1 + 2x + 3x^2 + 4x^3 + \cdots.$$

Then

$$(1 - x)S = 1 + x + x^2 + x^3 + x^4 + \cdots = \frac{1}{1 - x}, \text{ for } -1 < x < +1.$$

Hence

$$S = \frac{1}{(1 - x)^2}.$$

II. SOLUTION BY ELIJAH SWIFT, Princeton University.

The series converges for $-1 < x < +1$ and may be integrated in the open interval -1 to $+1$.

Then $\int_0^x S(x)dx = x + x^2 + x^3 + \cdots$, which is equal to $\frac{x}{1 - x}$. Differentiating this, we obtain $\frac{1}{(1 - x)^2}$ as the sum of the given series.

III. SOLUTION BY J. BROOKS SMITH, Hampden Sidney, Va.

The following solution is given by CHARLES SMITH, *Treatise on Algebra*, ex. 1, p. 413.

$$S_{n+1} = 1 + 2x + 3x^2 + 4x^3 + \cdots + (n + 1)x^n,$$

$$(1 - x)^2 = 1 - 2x + x^2.$$

Hence,

$$(1 - x)^2 \cdot S_{n+1} = 1 + x^{n+1}[n - 2(n + 1)] + (n + 1)x^{n+2}$$

(all the other terms vanishing on account of the identity,

$$k - 2(k - 1) + k - 2 \equiv 0).$$

Hence

$$(1 - x)^2 S_{n+1} = 1 - (n + 2)x^{n+1} + (n + 1)x^{n+2};$$

whence

$$S_{n+1} = \frac{1}{(1 - x)^2} - \frac{(n + 2)x^{n+1} - (n + 1)x^{n+2}}{(1 - x)^2}.$$

When $n \doteq \infty$, $S_{n+1} \doteq S \doteq \frac{1}{(1 - x)^2}$ for $-1 < x < +1$.

Solved in various other ways by ELMER SCHUYLER, B. LIBBY, HORACE OLSON, F. M. MORGAN, A. L. MCCARTY, C. HORNUNG, A. M. HARDING, CLIFFORD N. MILLS, H. C. FEEMSTER, OSCAR SCHMIEDEL, and A. G. CARIS.

A solution of 396 was received from ELIJAH SWIFT too late for credit in the last issue.

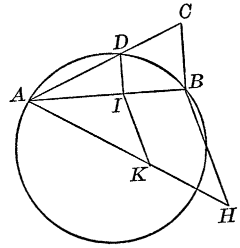
GEOMETRY.

428. **Proposed by R. D. CARMICHAEL, Indiana University.**

On a given chord of a circle as a base construct a right triangle with vertex outside of the circle such that its hypotenuse shall be bisected by its point of intersection with the circle. Are ruler and compasses sufficient to construct a triangle whose hypotenuse shall be thus divided in any ratio whatever?

SOLUTION BY EMMA M. GIBSON, Drury College.

The ruler and compasses are sufficient to construct a triangle whose hypotenuse shall be divided in any ratio as $m : n$, according to the conditions of the problem.



For let AB be the given chord. From A draw any line as AH and from A lay off $AK = m$ and $KH = n$. Then connect B and H and draw KI parallel to HB and intersecting AB in I. At I erect a perpendicular to AB intersecting the circle in D. Join A and D by a straight line and produce it until it meets a perpendicular erected at B in C. Then the right triangle ABC has its hypotenuse, AC, divided at D in the ratio of $m : n$.

For in the similar triangles AIK and ABH, $AI : IB = m : n$ and in the similar triangles ABC and AID, $AI : IB = AD : DC$. Hence, $AD : DC = m : n$.

When $m = n$, D bisects the hypotenuse.

Also solved by CLIFFORD N. MILLS, F. M. MORGAN, C. HORNUNG, HORACE OLSON, and S. W. REAVES.

429. Proposed by JOHN A. BIGBEE, Little Rock, Ark.

In the trihedral angle V-ABC, the face angle AVB is bisected by the straight line VD. Is it true that the angle DVC is less than, equal to, or greater than, half the sum of the angles AVC and BVC, according as $\angle CVD$ is less than, equal to, or greater than 90°?

SOLUTION BY A. M. HARDING, University of Arkansas.

(1) $\angle CVD < 90°$. (Fig. 1.) Take $VA = VB$. Join A and B. Let this line cut the bisector of $\angle AVB$ at D. Through D pass a plane perpendicular to

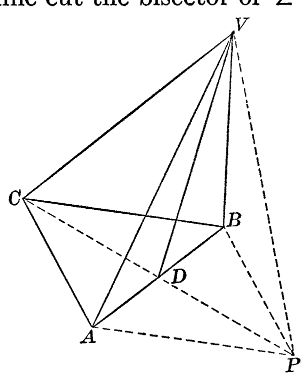


FIG. 1.

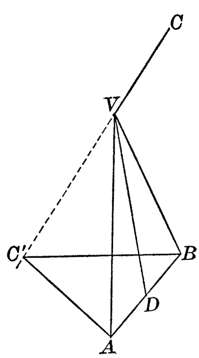


FIG. 2.

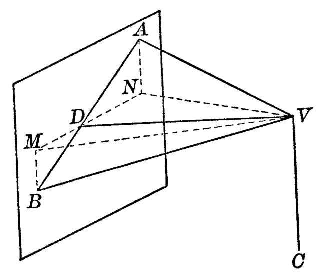


FIG. 3.